GROUND-WATER HYDRAULICS AS AN AID TO GEOLOGIC INTERPRETATION[1]

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The ground-water hydrologist has at his disposal a number of hydraulic tools that are of value in solving geologic problems. Among these is the aquifer-hydraulics test wherein the influence of pumping, or recharging, a well at a known constant rate is measured at several observation wells tapping the aquifer. From the results of such a test, the water-bearing properties of a rock unit can be evaluated and the location and hydrologic importance of its stratigraphic or structural boundaries can be appraised.

Rocks of the earth's crust serve as vast underground reservoirs for the storage and transmission of ground water. The physical properties and the hydrologic dimensions of the reservoirs differ according to the various geologic processes which contributed to their existence.

Interstices, or openings, in rocks differ widely in character, size and shape. Most are relatively small and permit only slow percolation of ground water. The principles of the occurrence of ground water are discussed in detail by Meinzer (1923) and the reader interested in further details is advised to consult this paper.

THE COEFFICIENT OF PERMEABILITY

The capacity of a formation to transmit ground water is indicated by its coefficient of permeability and is dependent upon the size, shape, arrangement, and assortment of materials, and upon the degree to which these materials have been compacted, cemented, indurated, or otherwise altered by geologic processes.

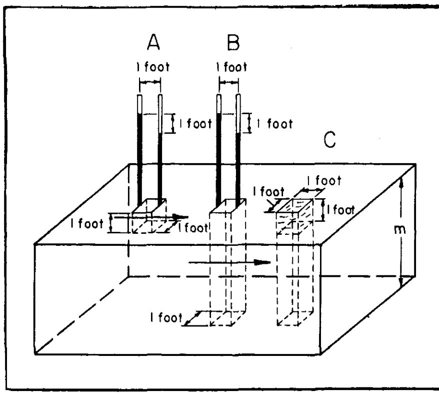
Field measurements of the permeability of consolidated rocks and unconsolidated deposits are made by means of aquifer-hydraulics tests. Field tests have an advantage over laboratory determinations, because aquifers are appraised in an undisturbed state, whereas laboratory permeability tests are dependent upon results obtained from small cores, or even disturbed samples, of materials which may not be truly representative of the aquifer being tested.

The size of the portion of an aquifer sampled by means of a single aquifer-hydraulics test depends upon the water-bearing properties, the geologic and hydraulic boundaries of the aquifer, and the duration of the test. In an artesian aquifer a 24-hour test may sample a cylinder of water-bearing material having a diameter of 1 mile or more. Under water-table conditions a 24-hour test may sample a cylinder having a diameter of 2,000 ft., or even less. Tests of more than 24 hours duration at several sites are desirable to evaluate properly the hydraulic properties of an aquifer.

The most productive aquifers in Ohio are the glacial valley-train and outwash-plain deposits of sand and gravel. It is possible that the over-all permeabilities of the valley-train deposits differ from valley to valley because of the variable distribution of glacial melt waters to the drainage basins in Ohio. The permeability of the outwash deposits is related to the amount of melt water that was available to rework and sort glacial materials. The permeabilities of valley-train and outwash-plain deposits also change from place to place within a given deposit as a result of variable conditions of deglaciation. Two or more outwash fills separated

[1]Presented at the meeting of the Ohio Academy of Science, Columbus, Ohio, on April 24, 1953. Publication authorized by the Director, U. S. Geological Survey.

by till layers resulting from repeated glaciation can be recognized in many areas in Ohio. The permeabilities of these separated deposits differ in many cases. The results of aquifer-hydraulics tests at several sites provide a means for detecting changes in the permeability of an aquifer, from which knowledge of the subsurface geology of an area may be gained and the geologic history inferred.



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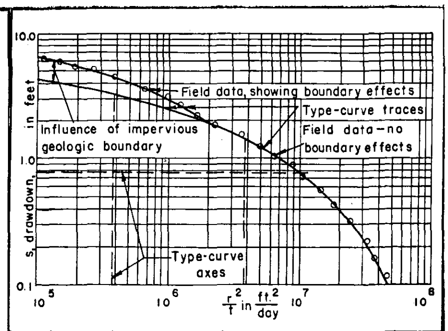

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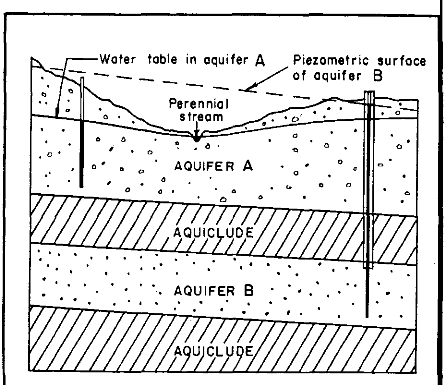

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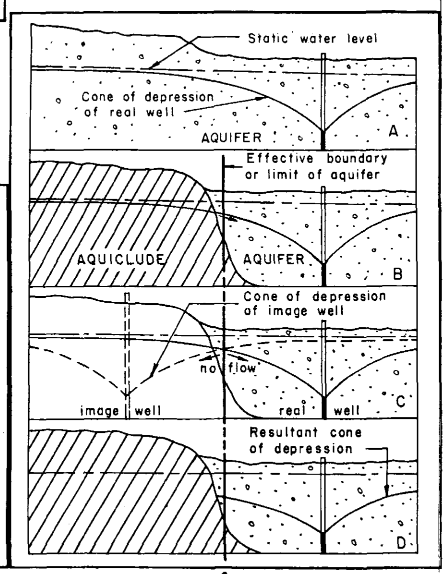

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FIGURE 1. Diagrammatic representation of the coefficients of permeability (A), transmissibility (B), and storage (C).
FIGURE 2. Diagrammatic representation of water-table and artesian conditions.
FIGURE 3. Log graph of aquifer-hydraulics test data.
FIGURE 4. Diagrammatic representation of the image-well theory.

The field coefficient of permeability, P, is defined as the rate of flow of water in gallons per day through a cross-sectional area of 1 sq. ft. of the aquifer under a hydraulic gradient of one foot per foot at the prevailing temperature of the ground water. (The laboratory coefficient is the same except that it is corrected to 60° F, so that samples from different areas can be compared directly). Although a hydraulic gradient of 1 foot per foot is used for convenience in the definition, it is not representative of field conditions. Instead, the gradient is usually expressed in feet per mile for flow through a section 1 ft. thick and 1 mile wide. The coefficient of permeability is represented by diagram A on figure 1. In natural materials it may range from a small fraction of a gallon per day per square foot to several thousand gallons per day per square foot.

THE COEFFICIENT OF TRANSMISSIBILITY

The coefficient of transmissibility, T, is defined as the rate of flow of water in gallons per day through a vertical strip of the aquifer 1 foot wide and extending the full saturated thickness under a hydraulic gradient of 100 percent (1 foot per foot) and at the prevailing temperature of the water. The coefficient of transmissibility is represented by diagram B on figure 1. In aquifers capable of yielding usable supplies of water to wells, it may range from a hundred or a few hundred gallons per day per foot to several million gallons per day per foot.

$P = \frac{T}{m}$ or $T = Pm$, where P is the field coefficient of permeability, T is the coefficient of transmissibility, and m is the saturated thickness of the aquifer.

THE COEFFICIENT OF STORAGE

Water must be supplied from storage in the aquifer until the required hydraulic gradient from the recharge area is established, or natural discharge is salvaged at the point of withdrawal. Under artesian conditions water is derived from storage by the compaction of the aquifer and its associated beds, and by expansion of the water itself. Under water-table conditions ground water is derived from storage mainly by the dewatering of a portion of the aquifer, but the coefficient of storage (see below) includes not only the water drained by gravity from the dewatered volume, but also a small amount of water derived from the underlying, still saturated, zone as described for artesian conditions. Thus the amount of water yielded from storage under water-table conditions is generally many times larger than that yielded under artesian conditions. Artesian and water-table conditions are shown on figure 2.

The storage properties of an aquifer are expressed by its coefficient of storage, which is defined as the quantity of water, expressed as a fraction of a cubic foot, that is discharged from each vertical prism of the aquifer with a base area of 1 sq. ft. and height equal to that of the aquifer, when the water level falls 1 foot. The coefficient of storage, S, under water-table conditions is represented by diagram C on figure 1.

THE CONE OF DEPRESSION

Ground-water reservoirs under natural conditions attain a state of approximate equilibrium in which discharge balances recharge. Artificial extraction of water from the reservoir, such as pumping from a well, disturbs the natural state of equilibrium. As pumping continues, a cone of depression with its center at the pumped well spreads out from the well in all directions until either (1) a hydraulic gradient is established from the recharge area of the aquifer to the pumped well sufficient to bring from the recharge area the amount of water being pumped, or (2) sufficient water is diverted from an area of natural discharge to balance the pumpage. The dimensions of the cone of depression depend upon the rate of recharge, the water-bearing properties and extent of the aquifer, and the rate and duration of pumping.

THE NONEQUILIBRIUM FORMULA

The formula used most widely to determine the coefficients of transmissibility and storage of an aquifer is the nonequilibrium formula (Theis, 1935). The nonequilibrium formula was derived by analogy between the flow of ground water and the flow of heat by conduction. Later Jacob (1940) derived this formula using hydraulic principles only. Well-hydraulics formulas developed prior to the nonequilibrium formula were based on the assumption that pumping had continued long enough to establish steady-state conditions. The Theis nonequilibrium formula first introduced the time factor and the coefficient of storage.

The nonequilibrium formula is

$$s=\frac{114.6\ Q}{T}\int_{\frac{1.87\ r^2S}{T\,t}}^{\infty}\frac{e^{-u}du}{u} \tag{1}$$

$$\text{where } u=\frac{1.87\ r^2S}{T\,t} \tag{2}$$

s = drawdown in feet at observation well
Q = discharge of pumped well in gallons per minute
r = distance in feet from pumped well to observation well
t = time in days since pumping began
T = coefficient of transmissibility in gallons per day per foot
S = coefficient of storage

The formula was developed on the basis of the following assumptions: that the aquifer is infinite in areal extent and is of the same thickness throughout, that it is homogeneous and isotropic (that is, its transmissibility is constant at all places and in all directions), that it is confined between impermeable beds, that the discharge of the pumped well is constant, that the coefficient of storage is constant, and that water is released from storage instantaneously with a decline in artesian head.

The value of the exponential integral $\int_{u}^{\infty}\frac{e^{-u}du}{u}$ is written symbolically as W(u) and can be computed by the following series:

$$W(u)=-0.577216-\log_e u+u-\frac{u^2}{2.2!}+\frac{u^3}{3.3!}-\frac{u^4}{4.4!}\cdots\cdots\cdots\text{etc.}$$

The nonequilibrium formula may be written as

$$s=\frac{114.6\ Q\ W(u)}{T} \tag{3}$$

W(u) is read "well function of u." Values of W(u) for values of u from 10^{-15} to 9.9 were published in a report by Wenzel (1942).

The presence of two unknowns and the nature of the exponential integral make an exact analytical solution impossible. T occurs twice in the equation and a solution by trial would be most laborious.

Theis devised a graphical method which makes solution of the equation relatively simple. Equations 2 and 3 are rewritten:

$$u=\left[\frac{1.87\ S}{T}\right]\frac{r^2}{t} \tag{4}$$

$$\text{and } W(u)=\left[\frac{T}{114.6\ Q}\right]s \tag{5}$$

The terms within the brackets are constant for a given aquifer and pumping rate. Equations 4 and 5 are similar and s is related to $\frac{r^2}{t}$ in the same manner that W(u) is related to u. If values of W(u) are plotted against corresponding values of u, and values of s are plotted against values of $\frac{r^2}{t}$, on cross-section arithmetic paper, the two curves will be similar but not identical in shape and curvature because the variables in the right halves of equations 4 and 5 are multiplied by different constants. Equations 4 and 5 may be rewritten:

$$\log u = \log \left[\frac{1.87\ S}{T}\right] + \log \frac{r^2}{t} \tag{6}$$

and

$$\log W(u) = \log \left[\frac{T}{114.6\ Q}\right] + \log s \tag{7}$$

In logarithmic form, multiplication becomes addition. A "type curve," representing the value of the exponential integral, is constructed by plotting values of W(u) against u on logarithmic paper. Values of s plotted on logarithmic paper against values of $\frac{r^2}{t}$ describes a "field-data curve" that is similar to the type curve. The difference in the constant multipliers of equations 4 and 5 cause relative displacement in the horizontal and vertical scales of the type curve and field-data curve plots. The field-data curve is superposed over the type curve. The coordinate axes of the two curves are held parallel and a match of the field-data curve to the type curve is obtained. In the matched position the vertical scales of the graphs are displaced with respect to one another by amount $\log \left[\frac{T}{114.6\ Q}\right]$ and the horizontal scales are displaced by the amount $\log \left[\frac{1.87\ S}{T}\right]$. A point on the type curve is selected and marked on the field-data curve. The coordinates of this common point, s, u, W(u), and $\frac{r^2}{t}$ are used in the nonequilibrium formula to determine the coefficients of transmissibility and storage. T is calculated using equation 3 with the W(u) and s coordinates. S is calculated using equation 2, the calculated value of T, and the $\frac{r^2}{t}$ and u coordinates of the match point. Figure 3 is a log graph of aquifer-hydraulics test data with a type curve superposed.

Details and variations of the methods available for determining the permeability of an aquifer are illustrated in the literature cited.

HYDROGEOLOGIC BOUNDARIES

The nonequilibrium formula was developed on the assumption that the aquifer being tested is infinite in areal extent. This condition of course is not satisfied in nature, although in large aquifers it may be satisfied for all practical purposes. Geologic conditions and surface hydrologic features limit most ground-water reservoirs to such an extent that their influence must be taken into account, at least in making long-term predictions of the effect of pumping. Hydrogeologic boundaries (Ferris, 1951), may consist of folds, faults, or impervious layers of shale or clay, or major changes in permeability caused by an increase or decrease in the number or size of pores or fractures, or the amount of included material of low permeability such as clay. Streams also form boundaries.

THE IMAGE WELL THEORY

The nonequilibrium formula may be used for the finite aquifer, if adjustment for the effect of hydrogeologic boundaries is made by means of the image-well theory. The hydrogeologic boundaries may be located from aquifer-hydraulics test data by utilizing the nonequilibrium formula and the law of times (described later in this report).

Most hydrogeologic boundaries are not clear-cut straight-line features but are irregular in shape and extent. However, because the area of the test site is relatively small compared to the areal extent of most aquifers, it is generally permissible to treat hydrogeologic boundaries as straight-line demarcations. Where this can be done, boundary problems can be solved by the substitution of a hypothetical hydraulic system that satisfies the hydrogeologic limits of the real system.

The method of images was devised by Lord Kelvin. The method of images as applied to ground-water hydrology may be stated as follows: The effect of an impervious geologic boundary (barrier) on the drawdown in a well as a result of pumping from another well is the same as though the aquifer were infinite and a like discharging well were located across the real boundary on a perpendicular thereto and at the same distance from the boundary as the real pumping well. For a boundary formed by a stream connected with the aquifer (line source) the principle is the same except that the image well is assumed to be recharging the aquifer instead of pumping from it.

For a demonstration of the image-well theory, consider an aquifer bounded on one side by an impervious formation. The impervious formation cannot contribute water to the pumped well. Water cannot flow across a line that defines the effective limit of the aquifer. The problem is to create a hypothetical infinite hydraulic system that will satisfy the boundary conditions dictated by the finite aquifer system.

Consider the cone of depression that would exist if the geologic boundary were not present, as shown by diagram A on figure 4. If a boundary is placed across the cone of depression, as shown by diagram B, the hydraulic gradient cannot remain as it was because it would cause flow across the boundary which is impossible. An imaginary discharging well placed across the boundary perpendicular to and equidistant from the boundary will produce a hydraulic gradient from the boundary to the image well equal to the hydraulic gradient from the boundary to the pumped well. A ground-water divide would exist at the boundary, as shown by diagram C, and this would be true everywhere along the boundary. The condition of no flow across the boundary line now has been fulfilled. Therefore, the imaginary hydraulic system of a well and its image counterpart in an infinite aquifer satisfies the boundary conditions dictated by the field geology of this problem. The resultant real cone of depression is the summation of the components of both the real and image well depression cones as shown by diagram D on figure 4.

When pumping begins, the water level in an observation well will draw down at an initial rate under the influence of the pumped well only. When the cone of depression of the image well reaches the observation well, the rate of drawdown will change. It will be increased in this instance because the total rate of withdrawal from the aquifer is now equal to that of the pumped well plus that of the discharging image well. Thus, the drawdown curve of the observation well is deflected downward.

If an aquifer-hydraulics test is conducted without prior knowledge of the existence of the impervious geologic boundary, it may be possible to locate the boundary by determining the position of the image well. The image well can be located by using data on the deflection of the drawdown curve under the influence of the discharging image well.

THE LAW OF TIMES

The law of times defined by Ingersoll, Zoebel, and Ingersoll (1948, p. 89) is

$$t_1/r_1^2 = t_2/r_2^2 = t_3/r_3^2 = \ldots\ldots\ldots\ldots t_n/r_n^2 \qquad (8)$$

The law of times as applied to ground-water hydrology may be stated as follows: For a given aquifer the times of occurrence of equal drawdown vary directly as the squares of the distances from an observation well to pumping wells of equal discharge. It follows, that, if the time intercept of a given drawdown in an observation well caused by pumping a well at a known distance is known, and if the time intercept of an equal amount of divergence of the drawdown curve caused by the effect of the image well is also known, it is possible to determine the distance from the observation well to the image well using this law of times.

Values of s and $\frac{r^2}{t}$ are plotted on log paper and the type curve is matched to the portion of the field-data curve that represents the effect of the real well only. The coefficients of transmissibility and storage are computed using these match-point coordinates and equations 2 and 3. The type curve is again matched to the field curve, this time over that portion of the drawdown curve influenced by the image well. A convenient intercept, r_p^2/t_i, is selected and the divergence of the two type-curve traces is noted. The intercept, r_p^2/t_p, of an equal drawdown on the earlier portion of the field-data curve is determined. The distance from the observation well to the image well is calculated using the following formula which expresses the law of times.

$$r_i = r_p \sqrt{\frac{r_p^2}{t_p} \Big/ \frac{r_p^2}{t_i}} \qquad (9)$$

r_i = distance in feet from observation well to image well

r_p = distance in feet from observation well to pumped well

t_p = time in days since pumping began for a particular value of s to be observed, before the boundary becomes effective

t_i = time in days since pumping began when the divergence of the drawdown curve from the type curve, under the influence of the image well, is equal to the value of s at t_p

Figure 3 is a log graph of aquifer-hydraulics test data showing the influence of an impervious geologic boundary.

If the distance to the image well from at least three observation wells can be calculated, the position of the boundary can be determined. The distances are scribed as arcs using the respective observation wells as centers; the intersection of these arcs locates the image well. The boundary is oriented perpendicular to, and crosses the midpoint of, a line joining the pumped well and the image well.

Aquifers limited in areal extent by two geologic boundaries can be analyzed by methods similar to those pertaining to a single boundary problem. Line-source boundaries formed by streams can also be located and analyzed from aquifer-hydraulics test data. These two problems, however, are beyond the scope of this paper.

The boundaries determined from aquifer-hydraulics test data represent the limits of a hypothetical aquifer system that is equivalent hydraulically to the real system. Although these effective boundaries do not completely describe the actual geologic boundaries of the aquifer, they do determine the location of the idealized geologic boundaries within reasonable limits. Aquifer-hydraulics tests, therefore, serve as aids to geophysical exploration and can greatly reduce the number of test wells required to confirm the existence of a geologic boundary (Ferris, 1951).

The collection of aquifer-hydraulics test data forms a part of water-resources investigations by the Ground Water Branch of the U. S. Geological Survey and its many cooperating State agencies. Several such tests are made each year in Ohio by the U. S. Geological Survey in cooperation with the Ohio Division of Water. Within a few years enough data will have been collected and analyzed to aid greatly in the interpretation of the geology of Ohio as it relates to the quantitative appraisal of the States' water resources.

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